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A N I N Q U I R Y

INTO THE PROBABLE

PARALLAX, AND MAGNITUDE

OF

T H E F I X E D S T A R S .

THE FIXED STARS
OF
PARALLAX AND MAGNITUDE

FROM
INTO THE PROBABLE
The Quantity of Light which they afford us and the
particular Circumstances of their Situation.
PARALLAX AND MAGNITUDE

By the Rev. JOHN HELL, B.D. F.R.S.



THE FIXED STARS



LONDON:
Printed in the Year MDCCCLVIII.

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INTO THE PROBABLE
PARALLAX, AND MAGNITUDE
OF
THE FIXED STARS,
FROM

The Quantity of Light which they afford us, and the
particular Circumstances of their Situation.

By the Rev. JOHN MICHELL, B.D. F.R.S.



L O N D O N:
Printed in the YEAR MDCCLXVIII.

A N I N Q U I R Y
 INTO THE PROBABLE
 PARALLAX AND MAGNITUDE
 OF THE FIXED STARS
 FROM

AN ENQUIRY
 INTO THE QUANTITY OF LIGHT WHICH WE RECEIVE FROM THE
 DISTANT CIRCUMSTANCES OF THEIR SITUATION.

ALTHOUGH no man can at present doubt
 that the want of sensible parallax in the
 fixed stars is owing to immense distance,
 yet it may not be unreasonable to see
 that this distance is far surpassed by other cir-
 cumstances; for let us suppose them to be, at a
 medium, equal in magnitude and natural bright-
 ness to the sun, to which they seem in all re-
 spects to be analogous. And, having laid this
 down as a foundation to build upon, let us inquire
 what would be the parallax of the sun, if he were
 to be removed so far from us, as to make the quantity
 of the light, which we should then receive from
 him, no more than equal to that of the fixed stars.
 In order to do this with accuracy, it would be proper to
 compare the quantity of light, which we at present
 receive from him, with that of the fixed stars, by
 some such method, as was made use of by Monsieur
 Bouguer.



Printed in the YEAR MDCCLXXII.

AN INQUIRY, &c.

THOUGH no man can at present doubt, that the want of a sensible parallax in the fixed stars, is owing to their immense distance, yet it may not perhaps be disagreeable to see, that this distance is farther confirmed by other circumstances; for let us suppose them to be, at a medium, equal in magnitude and natural brightness to the sun, to which they seem in all respects to be analagous. And, having laid this down as a foundation to build upon, let us inquire what would be the parallax of the sun, if he were to be removed so far from us, as to make the quantity of the light, which we should then receive from him, no more than equal to that of the fixed stars. In order to do this with accuracy, it would be proper to compare the quantity of light; which we at present receive from him, with that of the fixed stars, by some such methods, as are made use of by Monsieur Bouguer

Bouguer in his *Traité d'Optique*; but as my present purpose does not require any such exactness, I shall deduce it in a more gross way from facts already well known. I shall assume Saturn then in opposition, exclusively of his ring (and when the earth and he are at their mean distances from the Sun) as equal or nearly equal in light to the most luminous fixed star. Now the mean distance of Saturn from the Sun, being equal to about 2082 of the Sun's semidiameters, the density of the Sun's light, at Saturn, will consequently be less than at his own surface, in the proportion of the square of 2082 (or 4334724) to 1. If Saturn therefore was to reflect all the light, that falls upon him, he would be less luminous in the same proportion; but, besides this difference in his brightness, his apparent diameter, in opposition, is at most but 105th part of that of the Sun, and consequently the quantity of light, which we receive from him, must again be diminished in the proportion of the square of 105 (or 11025) to 1. If we multiply these two numbers together, we shall have the whole of the light of the Sun to that of Saturn (upon the supposition of his reflecting all the light, that falls upon him) as the square of nearly 220000 (or 48,400,000,000) to 1; and removing the Sun to 220000 times his present distance, he would still appear at least as bright as Saturn, and his whole parallax upon the diameter of the earth's orbit would be less than two seconds. This must consequently be assumed for the parallax of the brightest of the fixed stars, upon the supposition that their light does not exceed that of Saturn.

This work was published at Paris in 1760.

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By a like computation we shall find, that the distance, at which the Sun would afford us as much light, as we receive from Jupiter, is not less than 6000 times his present distance, and his whole parallax, in that case, upon the diameter of the earth's orbit would not be more than nine seconds, the light of Jupiter and Saturn, as seen from the Earth, being in the ratio of about 22 to 1, when they are both in opposition, and supposing them to reflect equally in proportion to the whole of the light that falls upon them.

But if Jupiter and Saturn, instead of reflecting the whole of the light, that falls upon them, should in fact reflect only a part of it, as for example, only a fourth or sixth (and this may very possibly be the case), we must then increase the distances computed above, in the proportion of 2 or 2 $\frac{1}{2}$ to 1, to make the Sun's light no more than equal to theirs; and his parallax would be less, in the same proportion, than those already mentioned.

Upon the supposition then, that the fixed stars are of the same magnitude and brightness with the Sun, it is no wonder, that their parallax should have hitherto escaped observation, since, if this is the case, it could hardly amount to two seconds, and probably

The light, which we receive from the full Moon (according to Monsieur Bouguer's experiments in the work above-mentioned), is only a 300000th part of that which we receive from the Sun, whereas it ought to amount to no less than a 45000th part of it, according to the principles, which we have made use of in computing the quantity of light derived from Jupiter and Saturn; so that the Moon, as appears from these experiments, reflects no more than between a sixth and a seventh part of the light that falls upon her.

not

not more than one in Sirius himself; though he had been placed in the pole^b of the ecliptic, and in those, that appear much less luminous, such for example as γ draconis, which is only of the third magnitude, it could hardly be expected to be sensible with such instruments as have hitherto been used in search of it.

We have assumed the magnitude of the fixed stars, as well as their brightness, to be equal to those of the sun; it is however probable, that there may be a very great difference amongst them in both these respects; and how much soever we may therefore be wide of the truth, in attempting to fix the distance of particular stars from this reasoning, yet there is a very great probability, that their mean distances, settled by this method, will not be much out, some exceeding and some falling short of it. And perhaps the consideration, that a star must be a thousand times as great, *ceteris paribus*, to appear equally bright, if it is placed at ten times the distance, may serve to make it probable, that the limits of the errors, which we are likely to commit, in judging by such a rule, are not so great as we might otherwise imagine them to be.

With regard to the difference there may be in the native brightness of different stars, though it is probably very considerable, yet I think we can hardly suppose, that it is equal to their difference in magnitude, at least if we except those, which are subject to certain changes, and which for that reason we may suppose to be luminous in some parts of their surfaces

^b The latitude of Sirius being only $39^{\circ} 33'$, his parallax will be a little less than two thirds of the whole parallax,

only.

only. In other instances we may perhaps judge in some degree of the native brightness of different stars with respect to one another by their colour; those, which afford the whitest light, being probably the most luminous.

* We have at present no means of judging of the comparative brightness of the Sun and of the fixed stars, in proportion to their respective sizes, excepting from the comparison of the Sun's brightness with that of our common fires; but the Sun's light exceeds the light of our brightest fires in so very great a proportion, (viz. of some thousands to one) that we want some middle terms to be able to form any analogy, which might serve to carry us farther. We find, however, in general, that those fires, which produce the whitest light, are much the brightest, and that the Sun, which produces a whiter light than any fires we commonly make, vastly exceeds them all in brightness; it is not therefore improbable, from this general analogy, that those stars, which exceed the Sun in the whiteness of their light, may also exceed him in their native brightness; now this is the case with regard to many of them; and, on the contrary, there are some that are of a redder colour.

It however it should hereafter be found, that any of the stars have others revolving about them (for no satellites shining by a borrowed light could possibly be visible), we should then have the means of discovering the proportion between the light of the Sun, and the light of those stars, relatively to their respective quantities of matter; for in this case, the times of the revolutions, and the greatest apparent elongations of those stars, that revolved about the others as satellites, being known, the relation between the apparent diameters and the densities of the central stars would be given, whatever was their distance from us: and the actual quantity of matter which they contained would be known, whenever their distance was known, being greater or less in the proportion of the cube of that distance. Hence, supposing them to be of the same density with the Sun, the proportion of the brightness of their surfaces, compared with that of the Sun, would be known from the comparison of the whole of the light which we receive from them, with that which we receive from the Sun; but, if they should happen to be either of greater or less density than the Sun, the whole of their light, not being

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As far then as we can guess at the parallax of the fixed stars from the principles above laid down, we may reasonably expect, that it should be exceedingly small even in those of the first magnitude; yet, besides the probability, that some of them may be either less, or less luminous than the Sun, it is not so small as to leave us altogether without hopes, that we may some time or other be able to discover it in some of them; for I think it is not impracticable to construct instruments, capable of distinguishing even to the 20th part of a second, provided the air will admit of that degree of exactness; but such instruments must be upon a plan a good deal different from those hitherto made use of, as they would otherwise be not only vastly too expensive, but also much too great and unweildy to be of any use.

But whatever room there may be to hope, that we may some time or other be able to discover the parallax of a few amongst the fixed stars, yet at the same time it seems probable, that we shall never be able to discover any sensible magnitude in their apparent diameters, which in Sirius himself, if he is not of less native brightness than the Sun, must be considerably less, whatever be his parallax, than the

affected by these suppositions, their surfaces would indeed be more or less luminous, accordingly as they were, upon this account, less or greater; but the quantity of light, corresponding to the same quantity of matter, would still remain the same.

The apparent distances, at which satellites would revolve about any stars, would be equal to the semiannual parallaxes of those stars, seen from planets revolving about the Sun, in the same periodical times with themselves, supposing the parallaxes to be such, as they would be, if the stars were of the same size and density with the Sun.

hundredth,

hundredth, probably than the two hundredth part of a second; so that it would scarcely be distinguishable with a telescope, upon the former supposition, that should magnify six, or, upon the latter with one, that should magnify twelve thousand times.

Nor can we well expect to find their apparent diameters from any occultation by the moon, since a diameter of the hundredth part of a second would be covered by the moon, if it entered directly, in less than the fiftieth part of a second of time, and therefore a star can hardly enter so obliquely, as to appear to vanish by degrees; no star probably, which the moon can pass over, subtending an angle half so great. A star might however appear to vanish by degrees in an occultation by the planet Venus, especially if the occultation was to happen only a little before or after either station; but this is an event, which can occur so very seldom, that little is to be expected from it; and if Venus should be surrounded with an atmosphere, which is probably the case, it might very possibly then be of no service at all. For the same reasons also it is probable, that nothing can be determined from occultations by any of the rest of the planets, which upon other accounts are still less proper for the purpose than Venus.

There seems to be little chance therefore of discovering with certainty the real size of any of the fixed stars, and we must consequently be contented to deduce it from their parallax (if that should ever be found) and the quantity of light which they afford us, compared with that of the Sun. And in the mean time, till this parallax can be found, or something else may arise to furnish us with a more general analogy,

we can only suppose them, at a medium, to be equal in size to the Sun, this being the best means, which we have at present of forming some probable conjecture concerning the extent of the visible universe. That we may be the better enabled to do this, it seems to be an object worth the attention of Astronomers, to enquire into the exact quantity of light, which each star affords us separately, when compared with the Sun; that, instead of distributing them, as has hitherto been done, into a few ill defined classes, they may be ranked with precision both according to their respective brightness, and the exact degree of it.

A catalogue of the stars formed upon this plan, would answer several good purposes; for, besides giving us a more just and certain notion of their general distances, it would perhaps help us, especially if the parallax of a few amongst them should be discovered likewise, to trace some analogies in their situation, which might enable us to determine both their distances, and other circumstances relating to them, with still more probability; and it would be a standing register, by which future Astronomers might inform themselves of many variations, of which we are now ignorant for want of an ancient register of that kind.

From what has been said above, I think it is very probable, that we shall not be a great way from the truth in estimating the whole parallax of Sirius at one second, supposing him to be of the same size and native brightness with the Sun; this therefore I shall assume as a standard, till some better experiments shall inform us more exactly of the quantity of his light.

light. Now, according to the best judgment I have been able to make from some gross experiments, the quantity of light which we receive from Sirius does not exceed the light which we receive from the least fixed stars of the sixth magnitude in a greater proportion than that of 1000 to 1, nor in a less proportion than that of 400 to 1; and the smaller stars of the second magnitude seem to be about a mean proportional between the two. Hence the whole parallax of the least fixed stars of the sixth magnitude, supposing them of the same size and native brightness with the Sun, should be from about $2''$ to $3''$, and their distance from about eight to twelve million times that of the Sun: and the parallax of the smaller stars of the second magnitude, upon the same supposition, should be about $12''$, and their distance about two million times that of the Sun.

I have hitherto argued about the distances of the fixed stars, upon the supposition of their being of the same size and brightness with the Sun; and, if this was really so, those which appear the brightest must be the nearest to us. That this is in general the case, I suppose, will be very readily allowed; for, though it is true, that a much greater degree of real magnitude may compensate for the greatness of distance, and there is no reason for assigning any one limit to them rather than another; yet, when it is as likely that the largest stars should be in any one part of space as in any other, the probability in favour of this hypothesis is very great: the real motions also, which have been observed amongst several of the brightest of the fixed stars, is another argument to the same purpose; and we shall find it still farther confirmed

confirmed by very strong arguments of analogy drawn from the circumstances of the particular situation of the stars in the heavens.

It has always been usual with Astronomers to dispose the fixed stars into constellations: this has been done for the sake of remembering and distinguishing them, and therefore it has in general been done merely arbitrarily, and with this view only; nature herself however seems to have distinguished them into groups. What I mean is, that, from the apparent situation of the stars in the heavens, there is the highest probability, that, either by the original act of the Creator, or in consequence of some general law (such perhaps as gravity) they are collected together in great numbers in some parts of space, whilst in others there are either few or none.

The argument, I intend to make use of, in order to prove this, is of that kind, which infers either design, or some general law, from a general analogy, and the greatness of the odds against things having been in the present situation, if it was not owing to some such cause.

Let us then examine what it is probable would have been the least apparent distance of any two or more stars, any where in the whole heavens, upon the supposition that they had been scattered by mere chance, as it might happen. Now it is manifest, upon this supposition, that every star being as likely to be in any one situation as another, the probability, that any one particular star should happen to be within a certain distance (as for example one degree) of any other given star, would be represented (according to the common way of computing chances) by a fraction, whose

whose numerator would be to it's denominator, as a circle of one degree radius, to a circle, whose radius is the diameter of a great circle (this last quantity being equal to the whole surface of the sphere) that

is, by the fraction $\frac{60^2}{6875.5^2}$, or, reducing it to a

decimal form, .000076154 (that is, about 1 in 13131) and the complement of this to unity, viz. .999923846,

or the fraction $\frac{13130}{13131}$, will represent the probability

that it would not be so. But, because there is the same chance for any one star to be within the distance of one degree from any given star, as for every other, multiplying this fraction into itself as many times as shall be equivalent to the whole number of stars, of not less brightness than those in question, and

putting n for this number, $.999923846^n$, or the fraction

$\frac{13130^n}{13131^n}$ will represent the probability, that no one of

the whole number of stars n would be within one degree from the proposed given star; and the complement of this quantity to unity will represent the probability, that there would be some one star or more, out of the whole number n , within the distance of one degree from the given star. And farther, because the same event is equally likely to happen to any one star as to any other, and therefore any one of the whole number of stars n might as well have been taken for the given star as any other, we must again repeat the last found chance n times, and

consequently

consequently, the number $.999923846$, or the fraction $\frac{231391}{13131}$ will represent the probability, that no where, in the whole heavens, any two stars, amongst those in question, would be within the distance of one degree from each other; and the complement of this quantity to unity will represent the probability of the contrary.

By a like reasoning, if we would compute the probability, upon the same supposition, that no two stars should be, one within the given distance x and the other within the given distance z of some one particular star, we must, first, find the probability, that no star, of the whole number of stars n , would be within the distance x from the given star, which will be represented, as before, by the fraction $\frac{6875.5 - xz}{6875.5}$

and, secondly, we must find the probability, that no star, of the whole number of stars n , would be within the distance z from the given star, which, for the same reason, will be represented by the fraction

$\frac{6875.5 - xz}{6875.5}$ and the complements of these to unity

will represent the respective probabilities of the contrary: but the probability that two events shall both happen, is the product of the respective probabilities of those two events multiplied together; if therefore we multiply the two last mentioned complements together, we shall have the probability, that some two stars would be, one within the distance x , and the other

other within the distance x from the given star; and the complement of this to unity, will be the probability, that it would not be so: let us now suppose $\frac{c}{d}$ to represent this last quantity, and, because the same event may as well happen in respect to any one star, as any other, multiplying this quantity into itself n times, according to the number of the stars, we shall have $\left(\frac{c}{d}\right)^n$ representing the probability, that no where, in the whole heavens, would be found any two stars, one within the distance x , and the other within the distance x from the same star.

If now we compute, according to the principles above laid down, what the probability is, that no two stars, in the whole heavens, should have been within so small a distance from each other, as the two stars β Capricorni, to which I shall suppose about 230 stars only to be equal in brightness, we shall find it to be about 80 to 1.

For an example, where more than two stars are concerned, we may take the six brightest of the Pleiades, and, supposing the whole number of those stars, which are equal in splendor to the faintest of these, to be about 1500, we shall find the odds to be near 500000 to 1, that no six stars, out of that number, scattered at random, in the whole heavens, would be within so small a distance from each other, as the Pleiades are.

* The computations of these probabilities are as follow.

The distance between the two stars β Capricorni is something less than $3\frac{1}{2}$; according to the rule above laid down, therefore, if we suppose 230 stars equal to these in brightness, the proba-

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If, besides these examples that are obvious to the naked eye, we extend the same argument to the smaller clusters, such for example as the Praesepe Cluster, that no two stars of that brightness will be found, any where in the whole heavens, within the distance of $3\frac{1}{3}$ from each other, will be represented by the fraction $\frac{6875.5}{3.33 \&c.}$ $\frac{230 \times 230}{4254603}$ From twice the Log. of 6875.5 [7.6746086] then, subtract twice the Log. of 3.33 &c. [1.0457496] and the remainder 6.6288590 will be the Log. of the number of times, that $3\frac{1}{3}$ &c. is contained in 6875.5 viz. 4254603 times, and consequently $\frac{230 \times 230}{4254603}$ will be equivalent to the former fraction. From the Log. of 4254602, subtract the Log. of 4254603, and the remainder will be $-.000000102$, the proportional part for an unit in the number 4254603: this multiplied into 230 times 230, or 52900, gives $-.0053958$, the Log. of the whole quantity, which corresponds to the proportional part for an unit between 80 and 81; this quantity therefore is equivalent to the fraction $\frac{80}{81}$ nearly, the complement of which to unity is $\frac{1}{81}$.

In the Pleiades, the five stars Taygeta, Electra, Merope, Alcyone, and Atlas are respectively at the distances of 11, 19 $\frac{1}{2}$, 24 $\frac{1}{2}$, 27, and 49 minutes from the star Maia nearly. From 7.6746086, twice the Log. of 6875.5, then, as before, subtract 2.0827834, twice the Log. of 11 (the number of minutes between Taygeta and Maia) and the remainder 5.5918232 will be the Log. of the number of times, that 11 is contained in 6875.5 viz. 390682 times; and consequently a fraction, whose denominator is this number, and whose numerator is this number less by an unit, multiplied into itself 1500 times, will represent the probability, that no star out of 1500, scattered by chance in the whole heavens, would be within the distance of 11 minutes from the star Maia. From the Log. of 390681 therefore subtract the Log. of 390682, as in the former example, and the remainder will be $-.00000111$, the proportional part for an unit in the number 390682, which multiplied by 1500 will give us $-.0016650$ for the Log. of the probability sought. In like manner from stars,

stars, as well those that are collected together in clusters, such for example, as the Præsepe Cancri, 7.6746086 subtract 2.5800692, twice the Log. of 19 $\frac{1}{2}$ (the number of minutes between Electra and Maia) and the remainder will be 5.0945394, the proportional part for an unit corresponding to the natural number of which will be—.0000349; and 1500 times this quantity, or—.0052350 will be the Log. of the quantity, representing the probability, that no star out of 1500 scattered by chance would be within the distance of 19 $\frac{1}{2}$ minutes from Maia. If we follow the same rule for the three remaining stars Merope, Alcyone, and Atlas, we shall find the similar Logs. corresponding to these to be—.0076650, —.0100395, and—.0330300 respectively. The natural numbers corresponding to these five Logs. taken in the same order, are .996172, .988018, .982506, .977148, and .926766, which severally express the respective probabilities, that no stars out of 1500 scattered by chance, would be within the same distances, at which the five stars above mentioned are found to be, from Maia. The complements of these quantities to unity .003827, .011982, .017494, .022852, and .073234, which severally express the respective probabilities, on the contrary, that such stars would be found within the distances above specified from the star Maia, must all be multiplied together, to determine the probability, that these events should all take place at the same time. The sum of the Logs. of these numbers is—9 + .1279139 or—8.8720861, which is therefore the Log. of all these numbers multiplied together; and subtracting this number from 0, or, what amounts to the same thing, changing the sign, we shall have the Log. of the number of times that this quantity is contained in unity, that is, about 744880000 times; a fraction, therefore, whose denominator is this number, and its numerator unity, will represent the probability in favour of all these events taking place together; and a fraction, whose denominator is the same number, and its numerator the same number less by an unit, will represent the probability of the contrary. But, as this event might as well have happened to any other star as to Maia, we must multiply this last fraction into itself 1500 times, according to the supposed number of stars, to find the probability, that it should not have happened any where in the whole heavens. Now the proportional part for an unit upon this number is—.000000005825, which multiplied

the nebula in the hilt of Perseus's sword; & as to those stars, which appear double, treble, &c. when seen through telescopes, we shall find it still infinitely more conclusive, both in the particular instances, and in the general analogy, arising from the frequency of them.

We may from hence, therefore, with the highest probability conclude (the odds against the contrary opinion being many million millions to one) that the stars are really collected together in clusters in some places, where they form a kind of systems, whilst in others there are either few or none of them, no whatever cause this may be owing, whether to their mutual gravitation, or to some other law or appointment of the Creator. And the natural conclusion from thence is, that it is highly probable in particular, and next to a certainty in general, that such double stars, &c. as appear to consist of two or more stars placed very near together, do really consist of stars placed near together, and under the influence of some general law, whenever the pro-

portion of brightness and upwards should be as by 1500 gives us — .00000087377, the proportional part for an unit in somewhat more than 496000.

But it must be observed, that this number is smaller than it ought to be upon two accounts; for, in the first place, this method of computation gives only the probability, that no five stars would be within the distance above specified from a sixth, if they occupied the largest space, they possibly could do, under that limitation; and secondly, we have made no allowance upon account of the different magnitudes, which, if it had been attended to, would have given a somewhat greater result. These considerations, however would have made the reasoning a good deal more intricate; and we have no need to descend to minutiae, a difference in the proportion of 10 to 1 not at all affecting the general conclusion.

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probability is very great, that there would not have been any such stars so near together, if all those, that are not less bright than themselves, had been scattered at random through the whole heavens. After what has been said, it will be natural to inquire, whether, if the stars are in general collected into systems, the Sun does not likewise make one of some system; and which are those, amongst the fixed stars, that belong to the same system with himself. Now, supposing the stars of one system to be, in general and at a medium, of the same size and brightness with those of another, the number of stars of any one apparent magnitude would bear the same proportion to the number of stars of any other apparent magnitude, as they would do, in case all the stars were scattered uniformly, and not in systems, provided the eye was not placed in or near to one of those systems. And, in this case, the brightness decreasing, as the square of the distance inversely, and the sphere, in which they are included, increasing, as the cube of the distance directly, the number of stars of any one degree of brightness and upwards, should be, as the cube of the square root of that brightness. Supposing then the faintest of the 2000 brightest stars to be less bright than the faintest of the first 70, in the proportion of about 30 to 1 (and I think the difference is not less than this) this number is smaller, than we might have expected, if the Sun was not one of a system, in the proportion of 2000 to about 12000 or 1 to 6; but I shall lay the less stress upon this argument, for want of a more certain determination of the proportion of light, which we receive from the stars of different magnitudes.

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If, however, upon a more accurate examination, it should be found, that the quantity of light above assigned is not far from the truth, or if the difference of light should be greater than I have supposed it to be (in which case the argument will be still stronger); this will add a considerable degree of weight to the other arguments drawn from analogy, in favour of the Sun's making one of a system of stars.

If we would now inquire, which are probably those stars, which compose part of the same system with the Sun; though it will not be possible to point them out with certainty, yet there are some marks, by which we may, with great probability include some and exclude others, whilst the rest remain more doubtful. Those stars which are found in clusters and surrounded with many others at a small distance from them, belong probably to other systems and not to ours. And those stars, which are surrounded with nebulae, are probably only very great stars, which, upon account of their superior magnitude, are singly visible, whilst the others, which compose the remaining parts of the same system, are so small as to escape our sight. And those nebulae, in which we can discover either none, or only a few stars, even with the assistance of the best telescopes, are probably systems, that are still more distant than the rest.

The Pleiades, as they appear to the naked eye, have been shewn above to be probably a system by themselves; and if we examine them still farther by means of telescopes, we shall find, that they are surrounded with so large a number of smaller stars, as to increase the odds against the contrary opinion many millions to one. Now supposing the Pleiades

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to be in reality a system of stars, the probability is at least, I suppose, an hundred to one, that no one amongst them, of those visible to the naked eye, belongs to the same system with the Sun; but that there are only such stars, as are greater than the rest. The exact quantity of this probability depends upon the number of stars, visible to the naked eye, belonging to this system; the proportion, that the space occupied by the Pleiades bears to the whole heavens; and lastly, how far the situation of any one of the Pleiades falls in with the general analogy of the stars composing this system, if any such general analogy should appear.

As the nebulae, and smaller constellations, composed of a great number of stars, within a small distance from one another, belong probably to other systems; so those, which being placed at greater distances from each other compose the larger constellations, and such as have few or no smaller stars near them, when examined with telescopes, belong probably to our own system. Most of the stars of the first and second magnitude have this criterion to distinguish them as belonging to the same system with the Sun, besides several other circumstances, such as their greater brightness; the proper motions*, that have

* The apparent change of situation, that has been observed amongst a few of the stars, is a strong circumstance in favour of those stars being some of the nearest to us. This apparent change of situation may be owing either to the real motion of the stars themselves; or to that of the Sun, or partly to the one, and partly to the other. As far as it is owing to the latter (which it is by no means improbable may in some measure be the case) it may be considered as a kind of secular parallax, which, if the annual parallax of a few of the stars should some time or other be discovered, and the quantity and direction of the Sun's motion been

been observed amongst some of them, their being more numerous, than we might naturally expect in proportion to the smaller stars, if they did not compose a part of the same system with ourselves, &c.

Besides the brighter stars, it is probable there are many of those of the smaller magnitudes also, that belong to the same system with the Sun. Now, if this should be the case, many of them being only fainter upon account of their less real magnitude, we may stand the same chance of finding a parallax amongst some of these, as amongst the brightest ones, provided we pitch upon such as happen to belong to our own system: to direct us with some probability to these, we have the circumstances above-mentioned of their composing larger constellations, and their having few stars lying very near to them; and perhaps there may be still a little more reason to suspect, that those stars form a part of the same system with ourselves, where, besides these circumstances, there have been observed changes of brightness, &c. amongst several of them in the same neighbourhood, such for instance as the changes, which have been observed amongst several of the stars in the constellations of the Swan, Cassiopeia, &c. We may, I think, also venture to add to these most of those stars, that are of a redder hue than the rest, as it is probable that they are a good deal bigger, in proportion to their brightness, than the whiter stars. (See the last note

should be discovered likewise, might serve to inform us of the distances of many of them, which it would be utterly impossible to find out by any other means.

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but two]. Many of them also have been observed to have a proper motion of their own, which with several other concurrent circumstances tends to make it highly probable, that they are some of the nearest to us.

Having thus endeavoured to establish the probability, that the Sun is one of a system of stars, placed by themselves in this part of the universe, I shall next inquire into some of the consequences of this hypothesis. Now if this is the case, and we suppose the whole number of stars, which belong to this system only (excluding those which belong to others), to amount to about 1000, we shall find it to be an even chance, that the parallax of the nearest amongst them does not exceed the parallax of one half that number in a greater proportion than that of 9 to 1, supposing the Earth to be placed in or near the centre of the whole system; nor in a greater proportion than that of about 12 to 1, supposing it to be placed very near the edge of the system; for supposing 1000 points to be placed within a sphere of any given magnitude, and that they are equally indifferent to every part of that sphere, if we assume any one of these points as a centre, we shall find, according to the known doctrine of chances, that there is an equal degree of probability whether any one of the rest shall or shall not fall within a sphere, described about the point assumed, of about seven ten thousandths of the size of the whole sphere; but the radius of such a sphere is about $\frac{1}{10}$ or a little less than $\frac{1}{10}$ of the radius of the whole sphere, that is about $\frac{1}{10}$ of the radius of a sphere of half the size of the greater one; and therefore a sphere, of about nine times this radius,

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will include half the greater sphere, if its centre be assumed near the centre of the greater sphere; and a sphere of twelve times this radius will include half the greater sphere, if the point be assumed almost in the surface of it.

If we assume the stars belonging to our own system to be about 1000; since they are in all respects similar to the Sun (excepting perhaps such amongst them as are liable to frequent changes), and we have nothing to determine us to one magnitude rather than another, we may most reasonably suppose the magnitude of the Sun to be a medium amongst the whole number. Upon this supposition, he will probably rank only with the stars of the fourth magnitude; and his light therefore, if he was removed to the medium distance of the other stars (viz. a distance equal to the radius of the sphere, that would include half the stars of our own system) would hardly, I think, exceed a 200th part of the light of Sirius; and consequently, if the parallax of Sirius would be about one second, if he was of the same size and native brightness with the Sun, it will be an equal chance, that the parallax of one half of the stars, belonging to this system, is not less than one second divided by the square root of 200, that is a little more than $4''$; and it will likewise be an equal chance, that the parallax of no one amongst them exceeds between 9 and 12 times that quantity, or a little more than two thirds of a second.

If, instead of 1000 stars, we suppose the whole number, belonging to this system, to be only about 350, the Sun will then, if he is of a medium size amongst them, rank probably with the stars of the third

third magnitude, and his light, at a medium distance, upon this hypothesis, would be, I apprehend, about a 50th part of that of Sirius. And therefore, according to the reasoning above, we should then find it an equal chance, that the parallax of one half of these 350 stars would not be less than about $8''$; and there would be the same chance, that the parallax of no one amongst them would be more than between $30''$ and about $1'$.

In the former supposition of 1000 stars; the apparent magnitude of the Sun, when removed to the medium distance, &c. it seems not improbable, that the biggest star in the system may perhaps exceed the Sun, in the proportion of about 1000 to 1; and in the latter supposition of 350 stars, &c. that it may perhaps exceed the Sun, in the proportion of about 120 to 1.

In whatever proportion the diameter of the Sun is greater or less than the medium we have taken for it in the suppositions above, in the same proportion will the parallaxes be increased or diminished; and in the inverse triplicate of that proportion must their magnitudes be diminished or increased.

Let us now examine the circumstances of the Pleiades; and, assuming the respective distances of the stars, composing that system, from each other to be, at a medium, equal to those of our own, let us see what will be the consequences of this supposition.

Now, if the Pleiades do not extend farther in the direction of a line drawn between the Earth and them, than in a direction at right angles to that line (which, from their composing a system, we have a right to suppose they do not) we can hardly allow the mean

distance of those, that are next to each other, amongst the fix stars visible to the naked eye, to be greater than what would subtend an angle, if seen directly from the earth, of about forty or fifty minutes. And consequently the distances between them and the earth would be about 70 times that distance, and their apparent brightness, seen from those that are next to each other, must be about 4900 times as great, as it appears to us. But, of the Pleiades, if I judge rightly, is not fainter than Sirius in a greater proportion than that of about 100 to 59; this star therefore must appear brighter to (the nearest of those fix) which are visible to the naked eye, than Sirius does to us, in the proportion of about 49 to 1. Let us now suppose all the stars belonging to the Pleiades, as well those discoverable with telescopes, as those which are visible to the naked eye, to be contained within a sphere, whose apparent diameter at the earth is two degrees; and consequently the mean distance of them from a spectator placed somewhere amongst them, as it might happen, would subtend an angle, when seen directly from the earth, of about a degree. Since therefore we have supposed the distances of the stars of our own system to be, at a medium, equal to those of the Pleiades; and consequently their mean distances from the earth to subtend at the Pleiades an angle of one degree, we shall have the distance of the Pleiades about 57 times as great as the mean distance of the stars of our own system, from the earth. Hence, if the biggest of the stars of our own system should be at this mean distance from us, and Sirius should be the biggest, of the Pleiades must exceed it in the proportion of about 200 to

to do for removing Sirius to 57 times his present distance, his light would then be fainter than it is, in the proportion of 3249 to 1, that is, fainter than of the Pleiades in the proportion of 32:49 to 1, supposing most of the Pleiades as above, to afford 122 hundredth part of the light of Sirius; but the magnitudes of stars, supposing them equally luminous, and their distance to be given, is as the cube of the square root of their brightness, and therefore of the Pleiades, upon this supposition, must be bigger than Sirius in the proportion of the cube of the square root of 3249 (that is 185) to 1. But I must observe, that according to general (and, I believe, I may say universal) analogy in all those nebulae, in which we discover stars bigger than the rest, these stars are placed towards the middle of their respective systems, and, if therefore the same thing obtains with regard to our own system, this will make 1 of the Pleiades still something greater than 1 of the stars of our own system. If the distance of the Pleiades is greater than the mean distance of the stars of our own system, in the proportion of 57 to 1, it would be necessary, in order to make stars, of the same real magnitude amongst the Pleiades, equally visible to us with those of our own system, to take in a pencil of rays of a greater diameter, than the pupil of the eye, in the same proportion: this, after proper deductions for the loss of light, could not well be effected by an object lens of less than two feet aperture. Now Dr. Hooke tells us, in his Micrographia, that with a telescope of twelve feet length, he discovered in the Pleiades 78 stars, and, with longer telescopes, many more, but

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if a telescope of twelve feet length, the aperture of whose object lens was probably less than two inches, increased the number of visible stars in the Pleiades to 78, we may well suppose, that with an object lens of two feet diameter, they would amount to more than 1000. What this number would be must depend however upon the gradation of real magnitude amongst the stars of that system, to which there must necessarily be some limit, and it is not therefore improbable, that observations of the increase of the number of stars amongst the Pleiades, &c. with telescopes of larger apertures, especially if this was carried on to very large sizes, might serve to inform us of many circumstances, both with regard to this gradation, and perhaps some other things, that would enable us to judge with more probability concerning the distances, magnitudes, &c. of the stars of our own system.

If we now imagine a spectator amongst the Pleiades to take a view of this system from thence, supposing the distance, as before, 57 times the mean distance of our own stars, we should appear to him as a nebula, in which there would be no star bright enough to be distinguishable by the naked eye; and with a telescope, the aperture of whose object lens was two inches, he would hardly, I suppose, be able to distinguish more than half a score stars at the utmost.

Having hitherto supposed the distances of the stars of our own system to be the same with those of the Pleiades, and examined the appearances according to that hypothesis, let us now, instead of their distances, suppose their magnitudes to be the same. This would make this system, as seen from the Pleiades, to subtend an angle of about twelve degrees instead of two,

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and about half a score of the biggest stars would be there visible to the naked eye; but a telescope, whose object lens was of two inches aperture, would in that case, I apprehend, take in almost all the stars belonging to this system of the fourth magnitude and upwards. These appearances fall in leis, I think, with the general analogy of what we see in the heavens, than the former supposition; but for want of more observations I cannot say this with any certainty; in the mean time however till we have something farther to go upon, it may not perhaps be amiss to take a kind of medium between the two, and suppose the Pleiades to be at about twenty times the mean distance of the stars belonging to our own system, in which case γ will exceed the biggest of these in the proportion of about eight or ten to one; or it will exceed the Sun, according to our former suppositions, of his being of a medium size amongst 1000 or 350 stars, in one case in the proportion of about eight or ten thousand, and in the other, about a thousand or twelve hundred to one; its parallax in the former case being about $26''$ and in the latter about $1''$.

I shall conclude this inquiry with one observation more, concerning the appearance of the stars of our own system, as seen from great distances. Whatever then the real distance and magnitude of these stars may be, provided we have not been greatly out in assigning the proportion of their light in respect to that of the Sun and one another, if they were to be seen from a distance, at which the whole system would not subtend an angle of more than six or eight minutes, it would appear only as a nebula, no single star being visible with perhaps any telescope, that has ever yet been

been made ; for at this distance, the distance between the earth and the biggest star of this system not subtending an angle of more than about three minutes (that is, about a twelve hundredth part of the radius) the stars of this system must appear less luminous than they do to ourselves, in the proportion of the square of 1200 (or 144,0000) to 1. And supposing the light of Sirius to exceed that of the least visible star in the proportion of about 1200 to 1, the brightest star therefore would still require to have its light increased in the proportion of 1200 to 1, before it would begin to be distinguishable. To do this would require a telescope, that should take in a pencil of rays of a larger diameter than the pupil of the eye, in the proportion of 35 to 1, that is, a pencil of about a foot diameter, exclusive of deductions; for the pupil of the eye is not less than a third of an inch in diameter, in a clear star-light night, when there is no Moon; but to obtain such a pencil, we must not make use of a refracting telescope (with two lenses only) of less than 15 inches, nor a reflector of less than nearly two feet aperture. This may serve to shew us, that those nebulae, in which we cannot distinguish any stars, may yet reasonably be supposed to consist of stars, though too far distant to be singly visible; since this would be the case with our own system, seen from as great a distance as we may well suppose those nebulae to be from us, if we judge of it from the magnitude of the visible area, which they occupy in the heavens.

Of the twinkling of the fixed stars.

Having never yet seen any solution of the twinkling of the fixed stars, with which I could rest satisfied,

fied*; I shall offer the following, which may not perhaps be found an inadequate cause of that appearance; at least it has undoubtedly some share in producing it, especially in the smaller stars.

It is not, I think, unreasonable to suppose, that a single particle of light is sufficient to make a sensible impression upon the organs of sight. Upon this supposition, a very few particles of light, arriving at the eye in a second of time, will be sufficient to make an object visible, perhaps not more than three or four; for though the impression may be considered as momentary, yet the perception, occasioned by it, is of a much longer duration: this sufficiently appears from the well-known experiment of a lighted body whirled round in a circle, which needs not make many revolutions in a second, to appear as one continued ring of fire. Hence then it is not improbable, that the number of the particles of light, which enter the eye in a second of time even from Sirius himself, may not exceed three or four thousand; and from stars of the second magnitude, they may therefore probably not much exceed an hundred. Now the apparent increase and diminution of the light, which we observe in the twinkling of the stars, seems to be repeated at not very unequal intervals, perhaps about four or five times in a second: why may we not then suppose, that the in-

* Some astronomers have lately adopted, as a solution of this appearance, the extreme minuteness of the apparent diameters of the fixed stars, which, they suppose, must, in consequence of this, be intercepted by every little mote, that floats in the air; but, that an object should be able to intercept a star from us, it must be large enough to exceed the apparent diameter of the star by the diameter of the pupil of the eye; so that, if the star was a mathematical point, it must still be equal in size to the pupil of the eye.

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equalities, which will naturally arise from the chance of the rays coming sometimes a little denser and sometimes a little rarer, in so small a number of them, as must fall upon the eye in the fourth or fifth part of a second, may be sufficient to account for this appearance? An addition of two or three particles of light, or perhaps of a single one upon twenty, especially if there should be an equal deficiency out of the next twenty, would I suppose be very sensible: this seems at least probable from the very great difference in the appearance of stars, whose light is much less different than, I imagine, people are in general aware of; the light of the middle-most star in the tail of the great Bear does not, I think, exceed the light of the very small star next to it, in a greater proportion than that of about sixteen or twenty to one; and Monsieur Bouguer tells us, in his *Traité d'Optique*, before mentioned, that he finds a difference in the light of objects of one part in sixty-six sufficiently distinguishable.

It will perhaps be objected, that the rays, coming from Sirius, are too numerous to admit of a sufficient inequality, arising from the common effect of chance, so frequently, as would be necessary to produce this effect, whatever might happen in respect to the smaller stars; but till we know what inequality is necessary to produce this effect, we can only guess at it either one way or the other; there is however another circumstance, that seems to concur in the twinkling of the stars, besides their brightness, and this is a change of colour. Now the red and blue rays being very much fewer, I apprehend, than those of the intermediate colours, and therefore much more liable to inequality from the common effect of chance, may help very much

much to account for this phenomenon, a small excess or defect in either of these making a very sensible difference in the colour.

It will now naturally be asked, why the frequency of the changes of brightness should not be often much greater, as well as sometimes less, than that above-mentioned, and why the interval of the fourth or fifth, or some such part, should be pitched upon, rather than the fortieth or fiftieth part of a second, or than a whole second, &c. for according to the length or shortness of the time assumed, the changes, that will naturally occur, from the effect of chance, will be smaller or greater in proportion to each other. The answer to this question will, I think, tend to render the above solution more probable, as well as to throw a good deal of light upon the whole subject. The lengths of the times then between the changes of brightness, if I am not mistaken, depend upon the duration of the perception before mentioned, occasioned by the impression of the light upon the eye, than which they seem to be neither much longer nor shorter. Whatever inequalities fall within a much shorter time than the continuance of this perception, will necessarily be blended together, and have no effect, but as they compose a part of the whole mass, but those inequalities, which fall in such a manner, as that they may be assigned to intervals nearly equal to, or something greater than the continuance of this perception, will be so divided by the imagination, which will naturally follow, and pick them out as they arise.

F I N I S.